

Follow Ideas Through Chapters 6-8

Chapter 6

Consider $\mu = E[X]$

$$\bar{X} \approx \text{Normal}(\mu, \sigma/\sqrt{n})$$

is Unbiased Estimator for $\mu = E[X]$
(and S^2 estimates σ^2)

If n is big (& σ unknown) then

$$\mu \approx \bar{x} \pm \frac{s}{\sqrt{n}} \cdot q_{\text{norm}}(\alpha/2)$$

is (1- α) Confidence Interval for μ

solve for α

$$2 \cdot p_{\text{norm}}\left(\frac{\bar{x} - \mu_0}{s/\sqrt{n}}\right) = \alpha$$

is p-value for testing against
 $H_0: \mu = \mu_0$

$$z_{\bar{x}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

is the "z-score"
of \bar{x} vs μ_0

$$z_{\alpha} = q_{\text{norm}}(\alpha)$$

is the " α -level cutoff"

Chapter 7

Chapter 8

If σ is unknown then standardize with S

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1) \approx \text{Normal if } n \geq 30 \text{ ("big")}$$

(1- α) CI is

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} \approx \pm q_t(\alpha/2, n-1)$$

solve for μ

If n is small (& σ unknown) then

$$\mu \approx \bar{x} \pm \frac{s}{\sqrt{n}} \cdot q_t(\alpha/2, n-1)$$

is (1- α) Confidence Interval for μ

solve for α

$$2 \cdot p_t\left(\frac{\bar{x} - \mu_0}{s/\sqrt{n}}, n-1\right) = \alpha$$

is p-value for testing against
 $H_0: \mu = \mu_0$

$$t_{\bar{x}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

is the "t-score"
of \bar{x} vs μ_0

$$n-1 = \# \text{"degrees of freedom"}$$

Population Proportion: $X \sim \text{Binomial}(n, p)$

$\hat{p} = X/n$ is Unbiased Estimator for p

$\hat{p} \approx \text{Normal}(p, \sqrt{pq/n})$ $\left\{ \begin{array}{l} \mu = p \\ \sigma = \sqrt{pq/n} \end{array} \right.$

$\left. \begin{array}{l} \hat{p} = x/n \\ \hat{q} = 1 - \hat{p} \end{array} \right\} \rightarrow p \approx \hat{p} \pm \sqrt{\hat{p}\hat{q}/n} \cdot z_{\text{norm}}(\alpha/2)$

is (1- α) Confidence Interval for p

2. $p_{\text{norm}}\left(\frac{\hat{p} - p_0}{\sqrt{p_0 q_0/n}}\right) \approx X P$

Note: Hyp. Test uses $p_0 q_0/n$ instead of $\hat{p}\hat{q}/n$

is p-value for testing against $H_0: p = p_0$

Note: p-value could also be computed more directly via pbinom

(1-tailed Hyp. Test) $\left\{ \begin{array}{l} p = p_{\text{binom}}(x, n, p_0) \quad \text{if } x < np_0 \\ 1 - p_{\text{binom}}(x, n, p_0) \quad \text{if } x > np_0 \end{array} \right.$

Variance $\sigma^2 = \text{Var}[X]$

$$S^2 = \frac{1}{n-1} \sum (\bar{X}_k - \bar{X})^2 = \frac{1}{n-1} \left[\sum \bar{X}_k^2 - \frac{1}{n} (\sum \bar{X}_k)^2 \right]$$

is Unbiased Estimator for σ^2

$$(n-1) \frac{S^2}{\sigma^2} \sim \chi^2(n-1)$$

"Chi Squared" with $n-1$ deg. of freedom

$$\frac{(n-1)S^2}{g_{\text{chisq}}(1-\alpha/2, n-1)} < \sigma^2 < \frac{(n-1)S^2}{g_{\text{chisq}}(\alpha/2, n-1)}$$

is (1- α) Confidence Interval

[We did not do Hypothesis Testing for Variance...]

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